Vanderbilt Center for Health Economic Modeling

Decision Trees & Probabilities

Back to Website

Learning Objectives and Outline

Back to Website

Learning Objectives

- Construct and solve a decision problem by calculating an intervention's **expected value** across competing strategies in a decision tree
- Determine the decision threshold across a range of scenarios
- Differentiate between joint and conditional probabilities and demonstrate their use in decision trees

Outline

1. Structure
2. Probability review
3. Probabilities within decision trees
4. Strengths/limitations of decision trees

Back to Website

The Structure of a Decision Analysis

Back to Website

Recap: Decision Analysis

- Aims to inform choice under uncertainty using an explicit, quantitative approach
- Aims to identify, measure, & value the **consequences of decisions** (risks/benefits) & uncertainty when a decision needs to be made, most appropriately over time

Should I go to the beach or stay home?

Back to Website

Should I go to the beach or stay home?

Possible States of the World:

- At the beach with no rain.
- At the beach with rain.
- At home with no rain.
- At home with rain.

Should I go to the beach or stay home?

Considerations:

- Likelihood of rain
- My overall well being when
  - At the beach with no rain.
  - At the beach with rain.
  - At home with no rain.
  - At home with rain.

Back to Website

Should I go to the beach or stay home?

Considerations:

- Likelihood of rain -> **probabilities**
- My overall well being when
  - At the beach with no rain.
  - At the beach with rain.
  - At home with no rain.
  - At home with rain.

Should I go to the beach or stay home?

Considerations:

- Likelihood of rain -> **probabilities**
- My overall well being when
  - At the beach with no rain. -> **payoff**
  - At the beach with rain. -> **payoff**
  - At home with no rain. -> **payoff**
  - At home with rain. -> **payoff**

VANDERBILT
Center for Health Economic Modeling

Decision Trees

- A square decision node indicates a decision point between alternative options.
- A circular chance node shows a point where two or more alternative events for a patient are possible.

Decision Trees

- A square decision node indicates a decision point between alternative options.
- A circular chance node shows a point where two or more alternative events for a patient are possible.

Decision Node

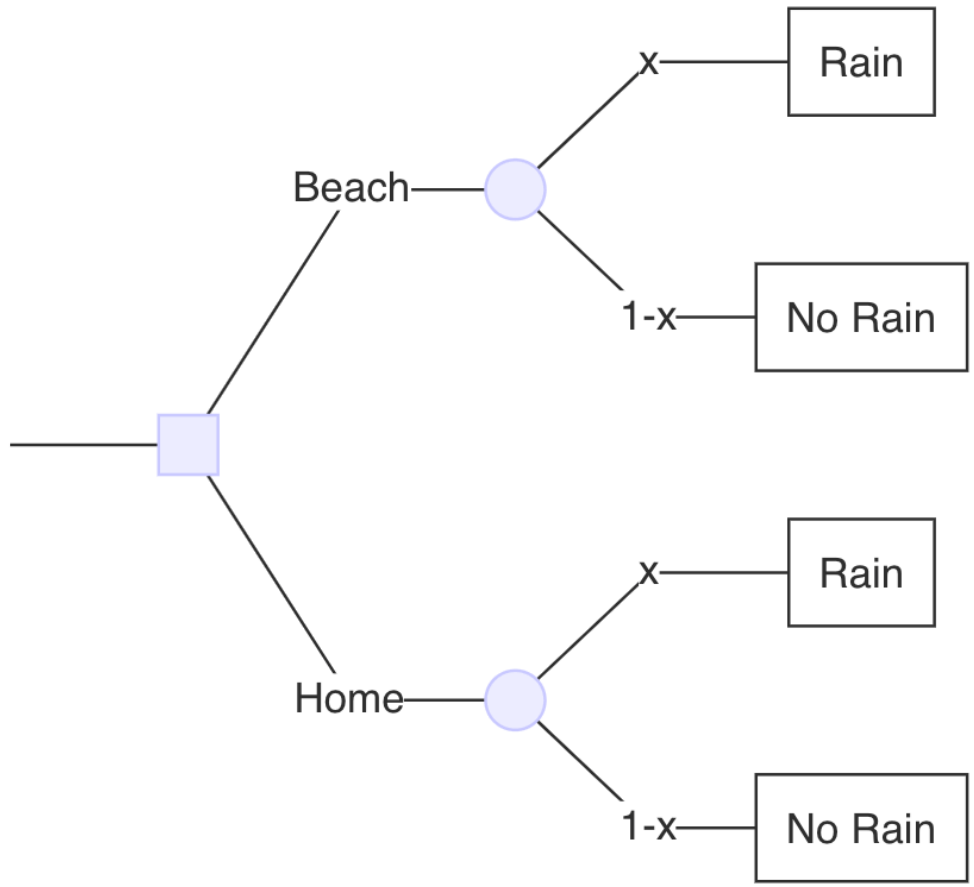
Chance Node

Decision Trees

- Pathways are mutually exclusive sequences of events and are the routes through the tree.
- Probabilities show the likelihood of particular event occurring at a chance node.

Should I go to the beach or stay home?

Decision Tree:



Back to Website

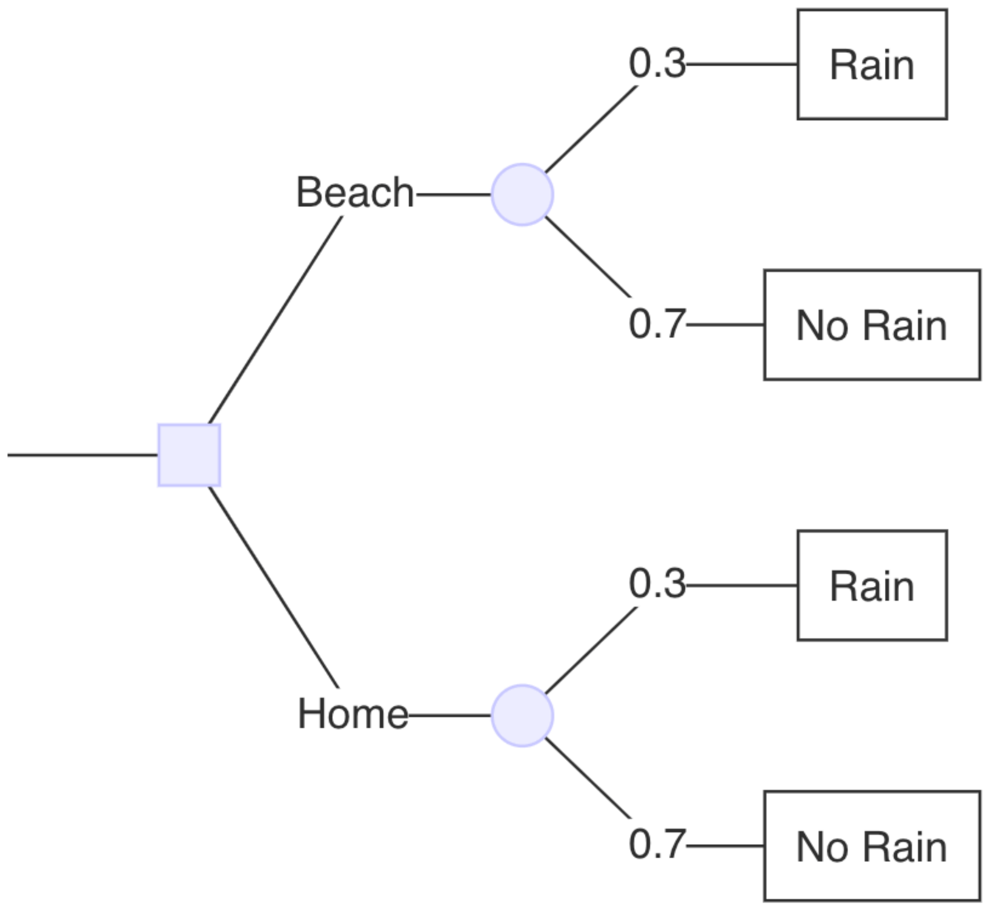
VANDERBILT
Center for Health
Economic Modeling

Probability of rain = 30%

Back to Website

Should I go to the beach or stay home?

Decision Tree:



Back to Website

Should I go to the beach or stay home?

Payoffs

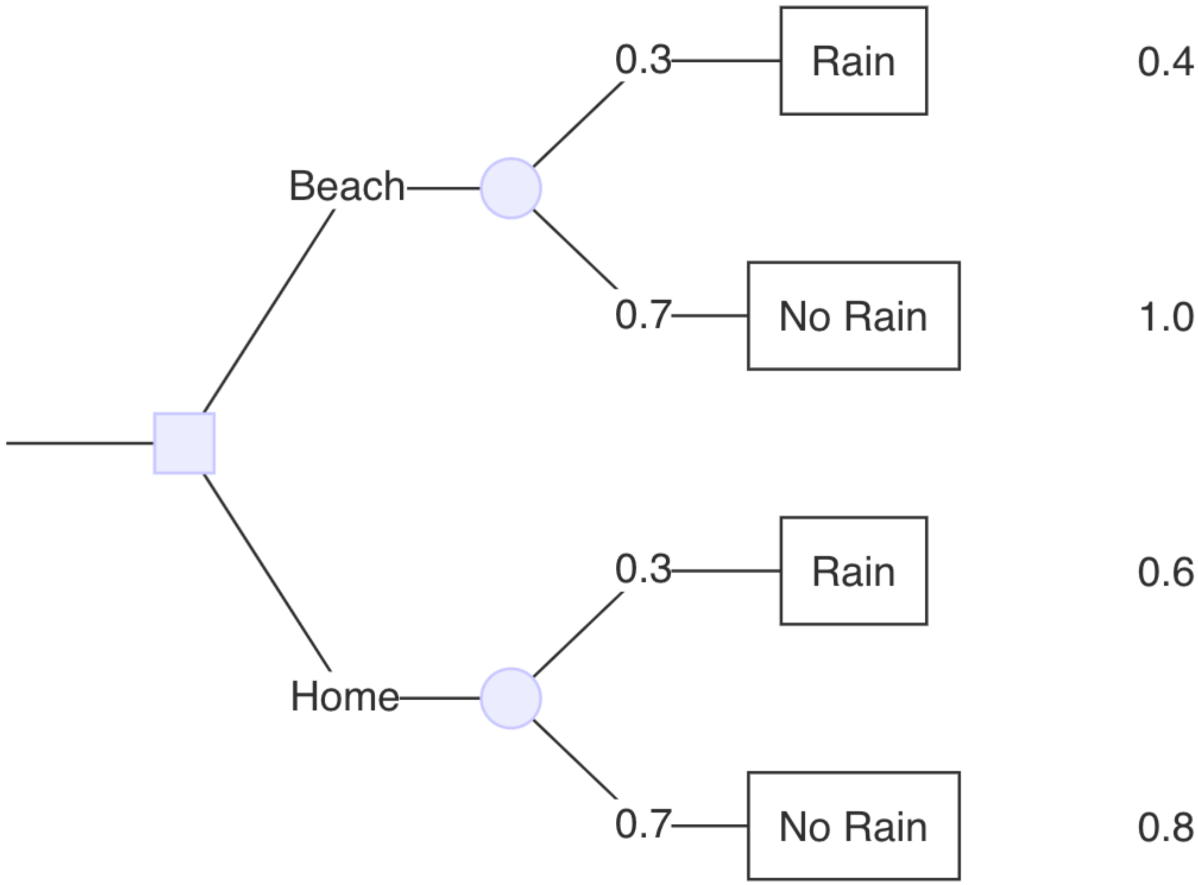
Scenario	Payoff
At beach, no rain	1.0
At beach, rain	0.4
At home, no rain	0.8
At home, rain	0.6

At beach, no rain > At home, no rain > At home, rain > At beach, rain

VANDERBILT
Center for Health
Economic Modeling

Should I go to the beach or stay home?

Decision Tree:



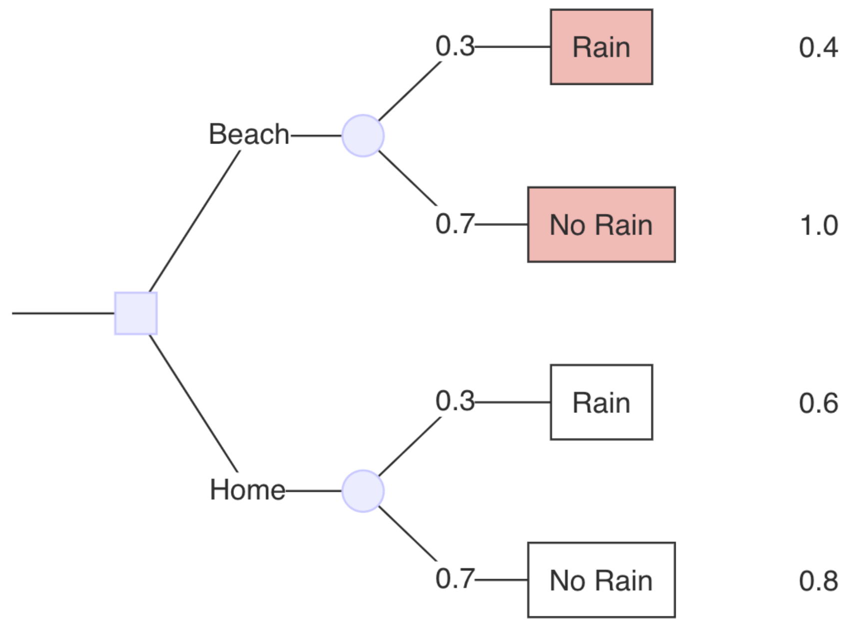
Back to Website

VANDERBILT Center for Health Economic Modeling

What is the expected value of going to the beach?

$$0.82 = \underbrace{0.3 * 0.4}_{\text{Rain}} + \underbrace{0.7 * 1.0}_{\text{No Rain}}$$

- Probabilities in red.
- Payoffs in blue.
- Expected value in green.



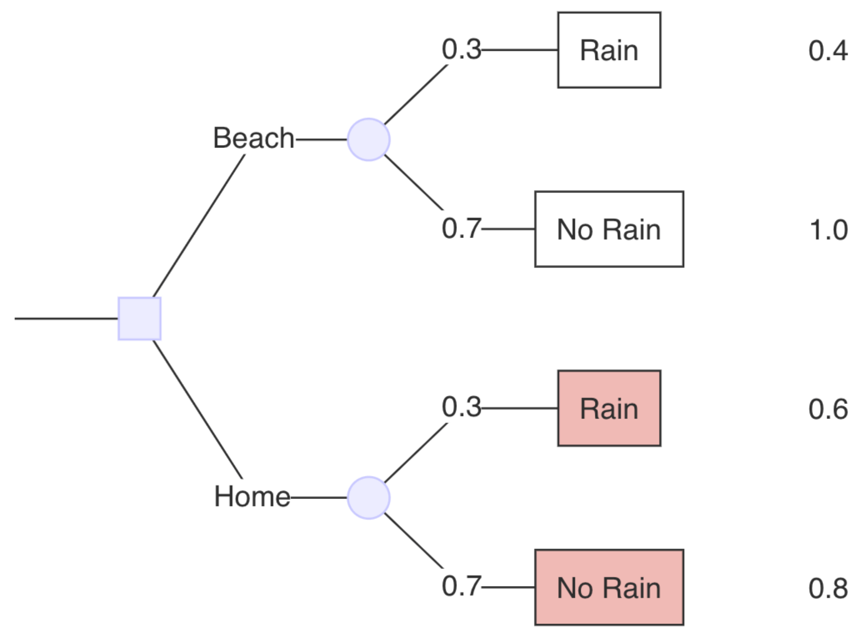
Back to Website

Vanderbilt Center for Health Economic Modeling

What is the expected value of staying home?

$$0.74 = \underbrace{0.3 \cdot 0.6}_{\text{Rain}} + \underbrace{0.7 \cdot 0.8}_{\text{No Rain}}$$

- Probabilities in red.
- Payoffs in blue.
- Expected value in green.



Beach

EV(Beach)=0.82 > EV(Home)=0.74

VANDERBILT Center for Health Economic Modeling

Expected Values

- Expected value = The sum of the multiplied probabilities for each chance option or intervention

$$E[x] = \sum_{x} {}_{i=1}^{\infty} \; X_i P(X_i)$$

VANDERBILT Center *for* Health Economic Modeling

Expected Values

- The expected value within the context of decision trees are the "payoffs" weighted by their preceding probabilities
- What we get is: the result that is expected ON AVERAGE for any one decision alternative (e.g. length of life, quality of life, lifetime costs)
  - Example: On average, patients given Treatment A will live 0.30 years longer than patients given Treatment B

> Maximizing expected value is a reasonable criterion for choice given uncertain prospects; though it does not necessarily promise the best results for any **one individual**

Determining the Decision Threshold

Back to Website

VANDERBILT Center for Health Economic Modeling

At what probability p are the two choices equal?

Suppose we want to know at what probability of rain p we are indifferent between going to the beach vs. staying at home…

- Write the equation for each choice using a variable, p, for the probability in question
- Set the equations equal to to one other and solve for p.

At what probability p are the two choices equal?

Beach: 0.82 = 0.3 x 0.4 + 0.7 x 1.0

Home: 0.74 = 0.3 x 0.6 + 0.7 x 0.8

At what probability p are the two choices equal?

$$\underbrace{0.3 * 0.4 + 0.7 * 1.0}_{\text{Beach}} = \underbrace{0.3 * 0.6 + 0.7 * 0.8}_{\text{Home}}$$

At what probability p are the two choices equal?

Replace probability of rain with **P** and **1-P** and solve for “P”

p * 0.4 + (1-p) * 1.0 = p * 0.6 + (1-p) * 0.8

$$\underbrace{0.3 * 0.4 + 0.7 * 1.0}_{\text{Beach}} = \underbrace{0.3 * 0.6 + 0.7 * 0.8}_{\text{Home}}$$

Back to Website

At what probability p are the two choices equal?

p * 0.4 + (1-p) * 1.0 = p * 0.6 + (1-p) * 0.8

VANDERBILT Center *for* Health Economic Modeling

At what probability p are the two choices equal?

p * 0.4 + (1-p) * 1.0 = p * 0.6 + (1-p) * 0.8

0.4p + 1-p = 0.6p + 0.8 - 0.8p

1-0.6p = 0.8 - 0.2p

1-0.8 = 0.6p - 0.2p

0.2 = 0.4 * p

0.5 = p

Back to Website

VANDERBILT Center *for* Health Economic Modeling

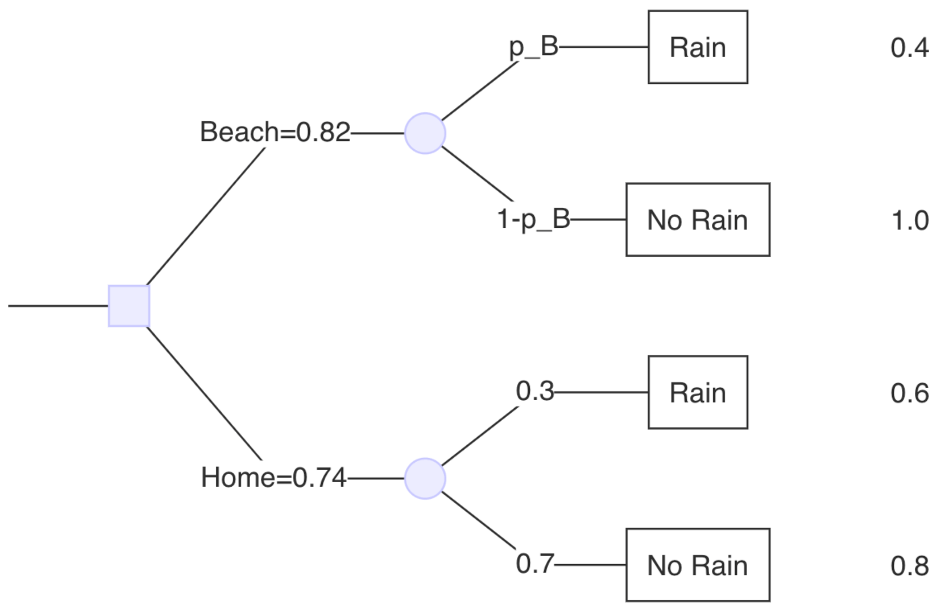
At what probability p are the two choices equal?

> When the probability of rain is 50% at BOTH the beach and home, given how we weighted the outcomes, going to the beach would be the same as staying at home

> In other words, you would be indifferent between the two – staying at home or going to the beach

VANDERBILT Center for Health Economic Modeling

At what probability (p_B) of rain for the beach are you indifferent between the two options?



Back to Website

VANDERBILT
Center *for* Health
Economic Modeling

At what probability (p_B) of rain for the beach are you indifferent between the two options?

- Earlier, we solved for the expected payoff of remaining at home: 0.74 (which was a lower expected value than going to the beach when the chance of rain at both was 30%)
- **What would** p_B need to be to yield an expected payoff *at the beach* of 0.74?
  - In other words, at what probability of rain at the beach would you be indifferent between staying at home & going to the beach?

At what probability (p_B) of rain for the beach are you indifferent between the two options?

Set 0.74 (expected value of remaining at home) equal to the **beach** payoffs and solve for p_B

p_B * 0.4 + (1 - p_B) * 1.0 = 0.74

At what probability (p_B) of rain for the beach are you indifferent between the two options?

$p_B * 0.4 + (1 - p_B) * 1.0 = 0.74$

$p_B * 0.4 + 1 - p_B = 0.74$

At what probability (p_B) of rain for the beach are you indifferent between the two options?

p_B * 0.4 + 1 - p_B = 0.74

p_B * -0.6 = -0.26

Back to Website

At what probability (p_B) of rain for the beach are you indifferent between the two options?

p_B * -0.6 = -0.26

p_B = -0.26 / -0.6 = **0.43**

At what probability (p_B) of rain for the beach are you indifferent between the two options?

> When the probability of rain at the beach is 43% (probability of rain at home remains at 30%), we would be indifferent between staying at home & going to the beach.

> If the probability of rain at the beach in > 43%, then we would stay home

Vanderbilt Center for Health Economic Modeling

Probability Review

Back to Website

VANDERBILT
Center *for* Health
Economic Modeling

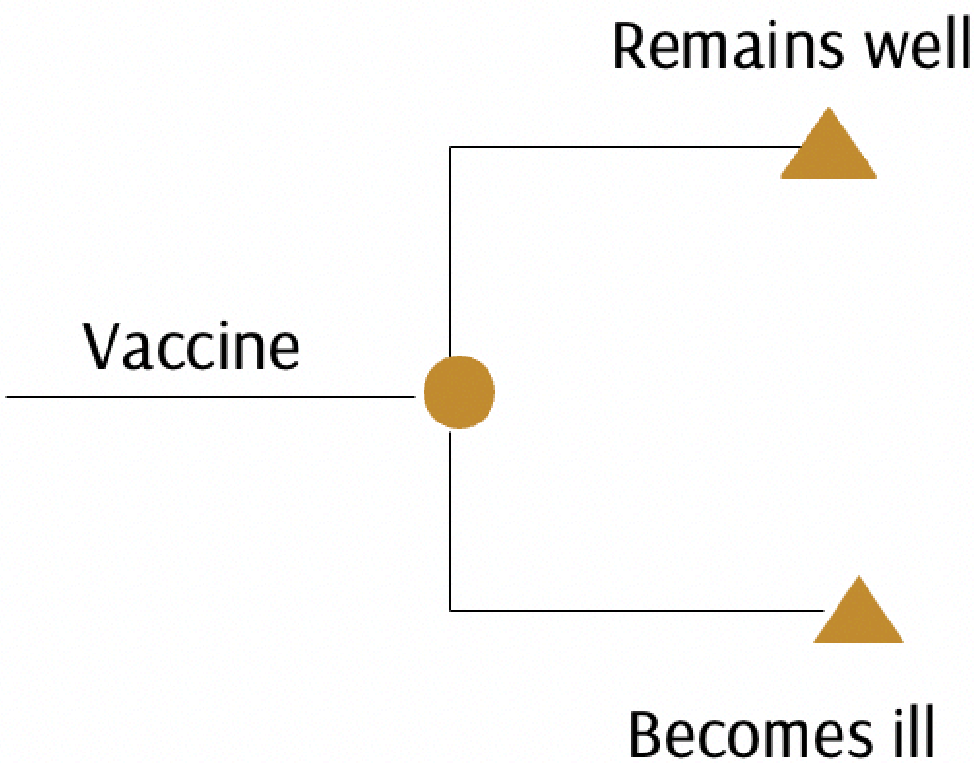
Probabilities within Decision Trees

Mutually exclusive events

- 2 things that cannot occur together (one event cannot occur at the same time as the other event)
  - Example: 2 events, survive or die; mutually exclusive because a person cannot be both at the same time
  - Example: 2 events, cured or not cured

Probabilities within Decision Trees

Mutually exclusive events



Back to Website

Probabilities within Decision Trees

Mutually exclusive events

- Assuming events are mutually exclusive, then the probability of 2 events occurring is the **sum of the probability of each event occurring individually**

$$P(A or B) = P(A) + P(B)$$

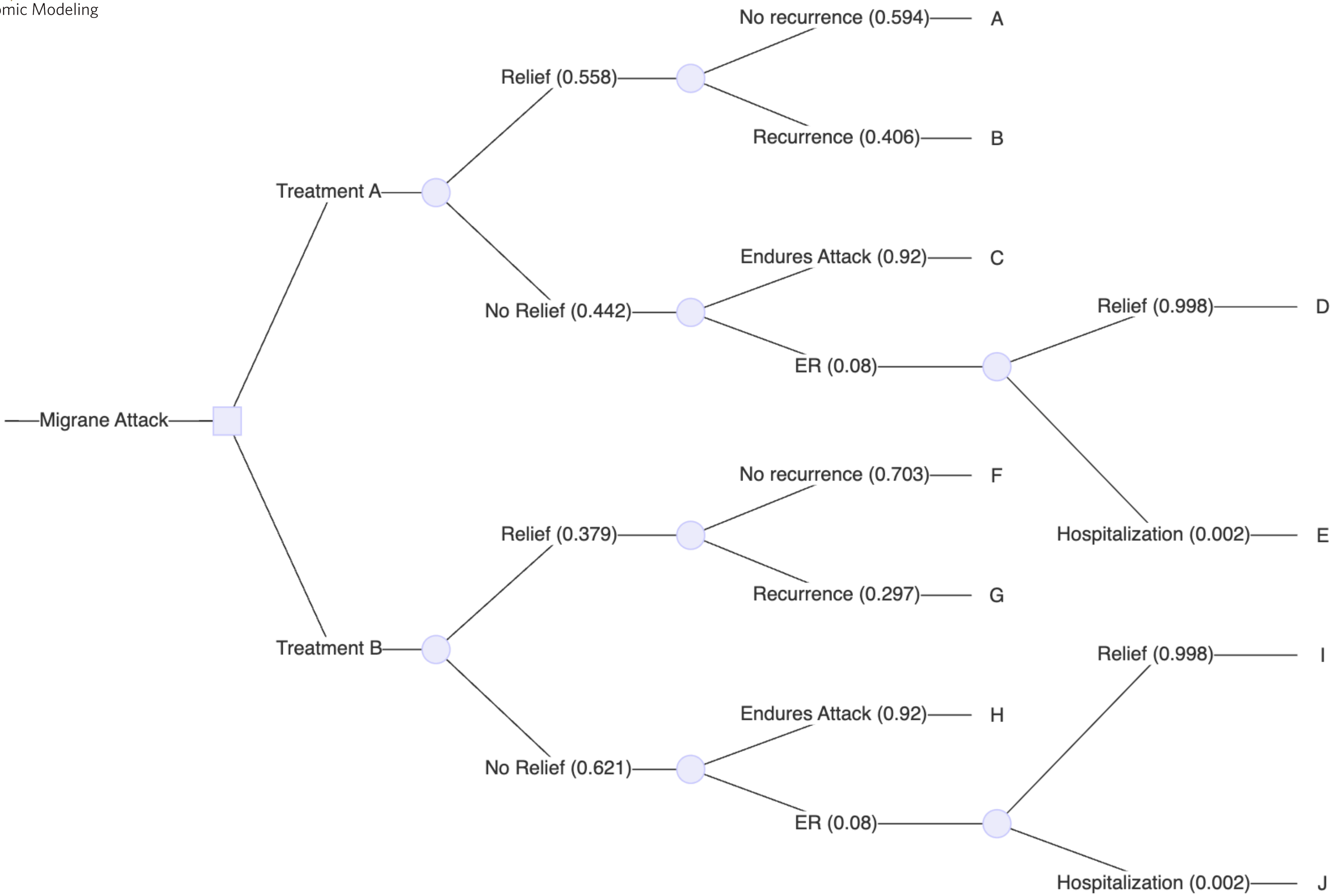
Probabilities within Decision Trees

Joint probability

P(A and B): The probability of two events occurring at the same time.

Conditional probability

P(A|B): The probability of an event A given that an event B is known to have occurred.



Back to Website

Probabilities

- Moving from left to right, the first probabilities in the tree show the probability of an event.
- Subsequent probabilities are **conditional.**
  - The probability of an event given that an earlier event did or did not occur.
- Multiplying probabilities along pathways estimates the pathway probability, which is a **joint** probability.

Pathways

- Pathways = sequence of events that lead to a subsequent "pay off"
- In other words, a sequence of events leads to an outcome/consequence, or payoff
- Example below: Our beach example has 4 pathways.

Pathways

Back to Website

VANDERBILT
Center for Health
Economic Modeling

Conditional probability

Example: What is the conditional probability of death within a year of birth, given the infant has a mother who smokes?

(Probability of an event occurring (B) given that another event occurred (A))

$$P(A|B) = P(AandB)/P(B)$$

VANDERBILT Center for Health Economic Modeling

Conditional probability, 2x2 review

What is the conditional probability of death within a year of birth, given the infant has a mother who smokes?

	Mother's smoking status		
Infant mortality	Smoker	Non-smoker	Total
Death	16,712	18,784	35,496
Live at 1-year	1,197,142	2,878,421	4,075,563
Total	1,213,854	2,897,205	4,111,059

**A= death in first year; B=mother who smokes*

P(A|B) = 16,712 / (1,197,142 + 16,712)
= 14 per 1,000 births

Back to Website

VANDERBILT
Center *for* Health
Economic Modeling

Conditional probability, 2x2 review

Or, if we wanted to use the conditional probability equation

	Mother's smoking status		
Infant mortality	Smoker	Non-smoker	Total
Death	16,712	18,784	35,496
Live at 1-year	1,197,142	2,878,421	4,075,563
Total	1,213,854	2,897,205	4,111,059

P(A|B) = P(A and B)/P(B)

**A= death in first year; B=mother who smokes*

P(A and B) = 16,712 / 4,111,059 = 0.0041

P(B) = 1,213,854/4,111,059 = 0.295

P(A|B) = 0.0041/0.295 = 0.014

Conditional probability, 2x2 review

Probability of A|B is different from that of B|A

	Birthweight		
Infant mortality	Low birthweight	Normal birthweight	Total
Death	21,054	14,442	35,496
Live at 1-year	271,269	3,804,294	4,075,563
Total	292,323	3,818,736	4,111,059

If A = death in first year; B=normal birth weight infant, then the conditional probability of P(A|B) = the probability of an infant death, given that the child has a normal birth weight

VANDERBILT
Center *for* Health
Economic Modeling

Conditional probability, 2x2 review

What is the conditional probability of an infant death, given that the child has a normal birth weight

	Birthweight		
Infant mortality	Low birthweight	Normal birthweight	Total
Death	21,054	14,442	35,496
Live at 1-year	271,269	3,804,294	4,075,563
Total	292,323	3,818,736	4,111,059

If A = death in first year; B=normal birth weight infant

$P(A|B) = 14{,}442 / (14{,}442 + 3{,}804{,}294)$

$= 3.8$ deaths per 1,000 births

[Back to Website]

Conditional probability, 2x2 review

Or, if we wanted to use equation

	Birthweight		
Infant mortality	Low birthweight	Normal birthweight	Total
Death	21,054	14,442	35,496
Live at 1-year	271,269	3,804,294	4,075,563
Total	292,323	3,818,736	4,111,059

P(A|B) = P(A and B)/P(B)

P(A and B) = 14,442

P(B) = 3,818,736

P(A|B) = 14,442/3,818,736 = 0.0038

Back to Website

VANDERBILT
Center for Health
Economic Modeling

Conditional probability, 2x2 review

On the other hand, the conditional probability of P(B|A) is the probability that an infant had normal birth weight, given that the infant died within 1 year from birth

	Birthweight		
Infant mortality	Low birthweight	Normal birthweight	Total
Death	21,054	14,442	35,496
Live at 1-year	271,269	3,804,294	4,075,563
Total	292,323	3,818,736	4,111,059

**B=normal birth weight infant; A = death in first year*

VANDERBILT
Center for Health
Economic Modeling

Conditional probability, 2x2 review

Solve P(B|A) – the probability that an infant had normal birthweight, given that the infant died within 1 year from birth

	Birthweight		
Infant mortality	Low birthweight	Normal birthweight	Total
Death	21,054	14,442	35,496
Live at 1-year	271,269	3,804,294	4,075,563
Total	292,323	3,818,736	4,111,059

P(B|A) = 14,442 / (14,442+ 21,054) = 0.41

Back to Website

Other Concepts in Decision Analysis

Back to Website

VANDERBILT
Center for Health
Economic Modeling

Decision Philosophies

> ! Maximizing expected value is a reasonable criterion for choice given uncertain prospects; though it does not necessarily promise the best results for any one individual.

- Mini-max regret
- Maxi-max
- Expected utility

VANDERBILT
Center *for* Health
Economic Modeling

Decision Philosophies

> ⓘ Maximizing expected value is a reasonable criterion for choice given uncertain prospects; though it does not necessarily promise the best results for any one individual.

- Mini-max regret
  - Never go to the beach unless 0% rain.
- Maxi-max gain
- Expected utility

Decision Philosophies

> Maximizing expected value is a reasonable criterion for choice given uncertain prospects; though it does not necessarily promise the best results for any one individual.

- Mini-max regret
  - Never go to the beach unless 0% rain.
- Maxi-max
  - Always go to the beach unless 100% rain.
- Expected utility

Back to Website

VANDERBILT
Center *for* Health
Economic Modeling

Decision Philosophies

Maximizing expected value is a reasonable criterion for choice given uncertain prospects; though it does not necessarily promise the best results for any one individual.

- Mini-max regret
  - Never go to the beach unless 0% rain.
- Maxi-max
  - Always go to the beach unless 100% rain.
- Expected utility
  - Depends on the weather.

Payoffs

- Each state of the world is assigned a cost or outcome.
- Our goal is often to calculate the expected value of these payoffs.
- We'll cover more on the theories and frameworks underlying various payoffs in the next few lectures

Strengths/limitations of decision trees

Back to Website

Strengths

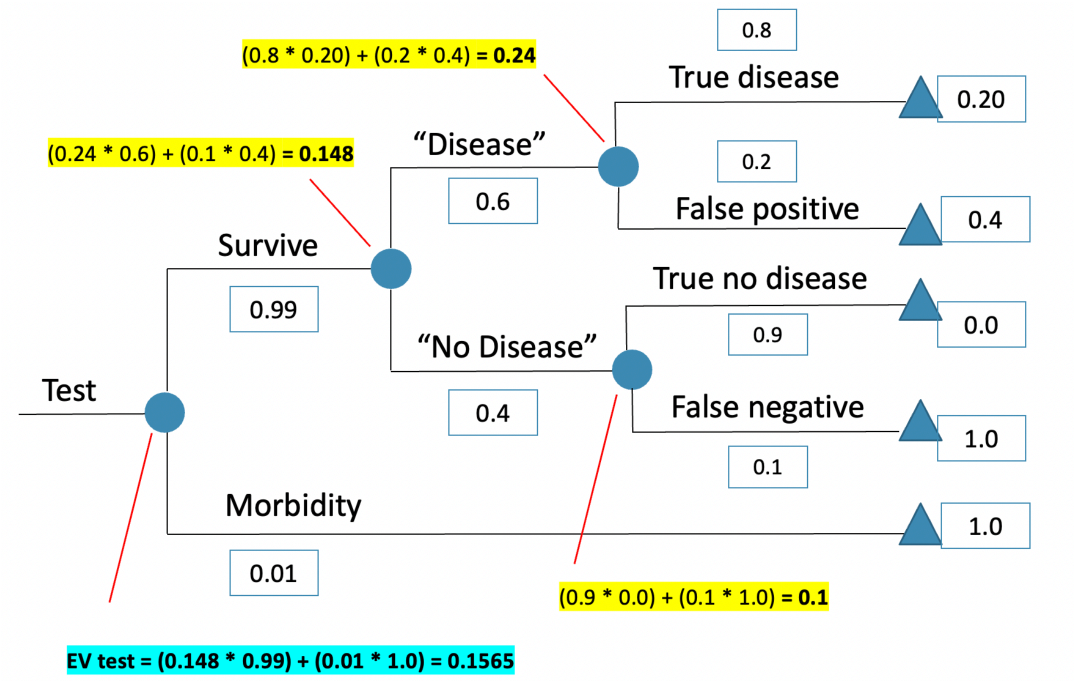
- They are easy to describe and understand
- Works well with limited time horizon
- Decision trees are a powerful framework for analyzing decisions and can provide rapid/useful insights, but they have limitations.

Limitations

- No explicit accounting for the elapse of time.
  - Recurrent events must be separately built into model.
  - Fine for short time cycles (e.g., 12 months) but we often want to model over a lifetime.
- Difficult to incorporate real clinical detail - Tree structure can quickly become complex.

VANDERBILT
Center for Health
Economic Modeling

Preview of what’s to come



- More complex tree!

Back to Website