Case Study: Probabilities & Decision Trees

> ❗ Important
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> Please note that you can download PDF and Microsoft Word versions of this case study using the links on the right.

Case 1

In a hypothetical community, 60% of all people consume at least 6 alcoholic beverages per week and 50% are overweight. The percentage of people who are both overweight and consume this much alcohol is 40%. Construct a 2x2 table to answer (a)-(c) below. For part (d), construct a decision tree.

- *What percentage of people consume at least 6 alcoholic beverages per week, are overweight, or fall into both categories?*

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	Overweight	Not overweight	Total
Drink 6 alcoholic beverages/week	0.4	0.2	0.6
Drink less or non-drinker	0.1	0.3	0.4
Total	0.5	0.5	1

P(Overweight OR Drinker) = 0.4 + 0.1+ 0.2 = 0.7
OR
P(Overweight OR Drinker) = 0.6 + 0.5 - 0.4 = 0.7
OR
1-neither (.03)

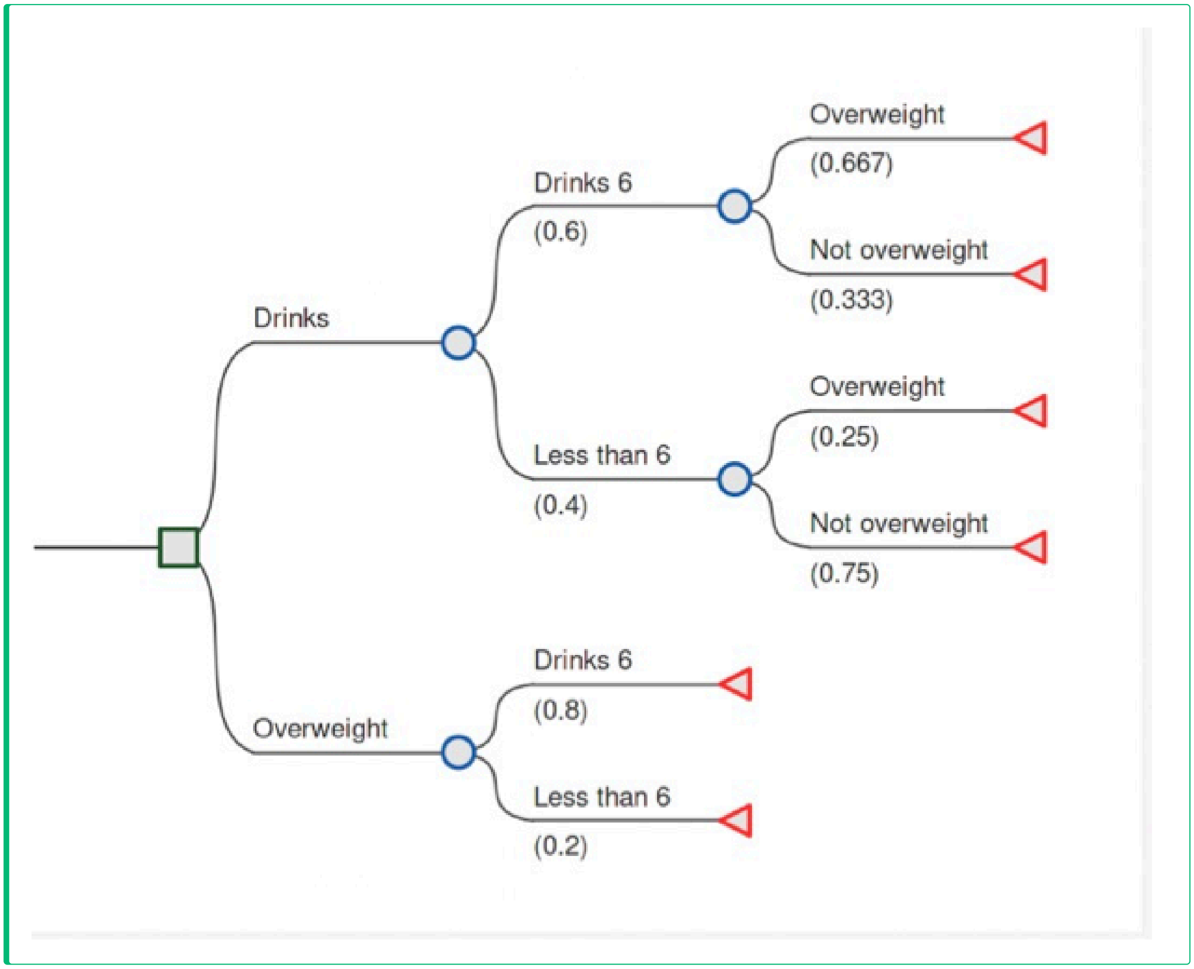
- *You sample at random a person from the community and find that they consume at least 6 alcoholic beverages per week. What is the probability that they are overweight?*

P(Overweight | Drinker) = 0.4 / 0.6 = 0.667

- *What is the probability that someone from this community consumes at least 6 alcoholic beverages per week if they are overweight?*

P(Drinker | Overweight) = 0.4 / 0.5 = 0.8

- *Draw a decision tree to represent this problem*



Case 2

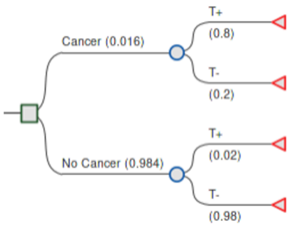
A new screening procedure can detect 80% of women diagnosed with breast cancer but will falsely identify 2% without breast cancer. The prevalence of breast cancer in the population is 1.6 in 100

- *What is the probability that a woman does not have breast cancer if the test is negative?*

*Note: There are two ways to do this – either draw a decision tree OR 2X2 table by creating a hypothetical population

(a)

P (C+ and T+) =. 0128
P(C+ and T-) = .0032
P(C- and T+)= .01968
P(C- and T-)= .96432
P(T+)= .0128+.01968=.03248
P(T-)= .0032+.96432=.96752



Then to solve for prob that woman does not have breast cancer given that test is negative:

P (C- | T-) = P(C- and T-)/P(T-)=.96432/.96752=.9967

- *What is the probability that a woman has breast cancer if the test is positive?*

(b) Prob that a woman has breast cancer given that the test is positive looks like this:

P(C+ | T+) = P(C+ and T+)/P(T+)=.0128/.03248= 0.3941

Another way to do it is to build a 2X2 table: since we know that the incidence of disease is 1.6 in 100, take a hypothetical population of 100,000 women

	Cancer +	Cancer -	
Test +	1,280 (1,600*.8)	1,968 (98,400*.02)	3,248
Test -	320	96,432	96,752
	1,600	98,400	100,000

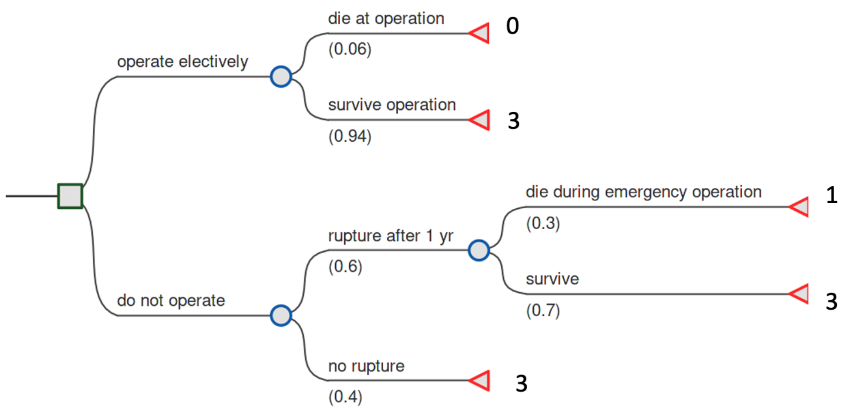
And now you can get the same probabilities above.

Case 3

A patient is found to have an abdominal aortic aneurysm (AAA) 5 cm in size. If you operate now and he survives, he will have a life expectancy of 3 additional years. In a series of 100 similar patients from your hospital, 6 died immediately after surgery. If you elect to watch the patient, 60% will rupture their AAA at home (assume at an average of 1 year later). Of those who rupture their AAA, 30% will die, while the other % will undergo emergency surgery and survive, allowing the patient to survive the full 3-years of life expectancy

- *Draw a decision tree for the problem of choosing whether to operate electively. Remember that you must consider the life expectancies as an outcome here. What is the preferred choice?*

(a) The following decision tree can be drawn for the problem:



- The life expectancy for 'operate electively' is: (0.06 * 0) + (0.94 * 3) = 2.82
- The life expectancy for 'do not operate' is: (0.4 * 3) + 0.6 * ((0.3 * 1 + 0.7 * 3)) = 2.64

The preferred choice is 'operate electively' since it has a higher life expectancy than 'do not operate'.

- *A 95% confidence interval for the mortality rate of elective surgery at your hospital ranges from 1.4-12.7%. Does this influence your thoughts? Why? Do you need better information about your estimate of mortality? Why? (Note: For the latter, calculate the mortality rate for which you are indifferent between the two choices)*

(b) The life expectancy for 'operate electively' with an operative mortality rate of 0.014 increases to (0.014 * 0) + (0.986 * 3) = 2.958. If you must choose between 'operate electively' and 'do not operate', you will still choose 'operate electively', since it has the highest life expectancy.

On the other hand, if you consider a mortality rate of 0.127 the life expectancy decreases to (0.127 * 0) + (0.873*3) = 2.619, and 'do not operate' will be the preferred strategy.

Based on these calculations you can say that the choice between 'operate electively' and 'do not operate' is sensitive to the mortality rate associated with the operation. You can calculate from the decision tree the mortality rate for which you are indifferent between the two choices:

2.64 = P * 0 + (1 – P) * 3
2.64 = (1 - P) * 3
2.64 = 3 - 3P
-3P = 2.64 - 3
-3P = -0.36
P = 0.12

Anything above this you wouldn't operate; anything below, you would operate. However, since the mortality rate was based on only 100 patients, the confidence interval was wide (& our decision changes within the CI). Gathering information based on more patients might be useful to determine whether the mortality rate is clearly above or below the threshold of 0.12.

Case 4

A patient presents to the ER with abdominal pain. As per the ER doc, you estimate that the patient's appendicitis probability is 0.16. If the patient truly has appendicitis, the probability that the appendix was already perforated at the time the patient presents to the ER is 0.1875. You can also observe the patient for 6 hours to be certain that your diagnosis is correct. If your diagnosis ends up being correct, 24% of individuals will have a perforated appendix after 6 hours (Note: this number is not what goes into your tree. You must account for the 18.75% who already had a perforated appendix at the time the patient entered the hospital).

If the appendix is perforated at the time the patient presents to the ER or at the end of 6 hours, there is a 0.84 chance that the symptoms will become worse and a 0.16 chance they will remain the same. If the patient has appendicitis but the appendix does not burst at the end of 6 hours, there is a 0.8 chance the symptoms will worsen and a 0.2 chance that they will remain the same. If the appendix is not diseased, there is a 0.39 chance that the symptoms will remain the same in 6 hours, a 0.61 chance that this will improve, and no chance that they will worsen.

- Calculate the probability that a patient has a perforated appendix by the end of 6 hours given that he had appendicitis but was not perforated at the time he entered the hospital

(a) The probability that a patient has a perforated appendix by the end of six hours given that the appendix was inflamed but not perforated at the time the patient entered the hospital:

$0.2400 = 0.1875 + (1 - 0.1875) * X \Leftrightarrow$
$X = (0.2400 - 0.1875) / (1 - 0.1875) \Leftrightarrow X = 0.0646$

You know that 24% will have a perforated appendix after six hours. 18.75% had a perforated appendix at the time the patient entered the hospital. 81.25% of the patients with appendicitis did not have a perforation at the time they entered the hospital, and they have a probability X of having a perforation in the next six hours.

Another way of thinking about it:

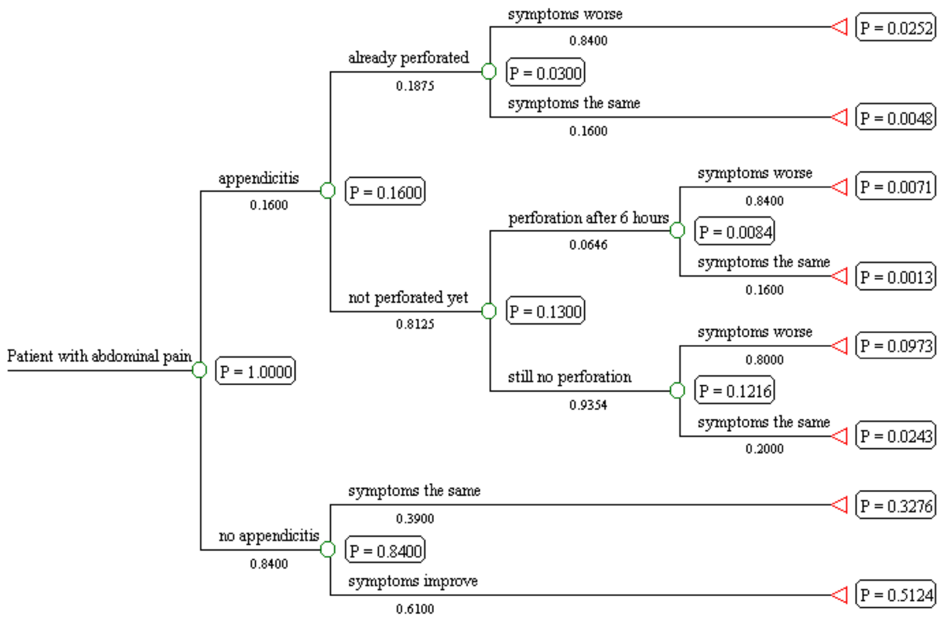
Let's say we have a population of 10,000 people. Based on the given probabilities, 1875 already have perforated appendices and 8125 do not. At the end of 6 hours, the number of perforated appendices will be 2400. The additional perforations come from the group of 8125. So:

P(Perforation after 6 hours | sick but not perforated yet) = (2400-1875)/8125 = **0.0646**

Draw a decision tree to calculate the probabilities asked in questions 3-6.

- *Calculate the probability that the patient has a perforated appendix at the beginning of 6 hours.*

(b) The following chance tree can be drawn for this problem. An easy way to answer the following questions is to calculate the probabilities at the end of each branch of the decision tree.



P(Perforation at the beginning of the six hours) = 0.1600 * 0.1875 = 0.0300. This is the proportion of patients with appendicitis multiplied by the conditional probability of perforation given appendicitis at the time the patient enters the hospital.

- *Calculate the probability that the patient will have a perforated appendix if you wait 6 hours*

P(Perforation after six hours) = 0.0300 + 0.1600 * 0.8125 * 0.0646 = 0.0384 P(Perforation after six hours) = 0.1600 * 0.2400 = 0.0384. This is the proportion of patients with appendicitis times the proportion of patients with a perforated appendix.

- *Calculate the probability that the patient's symptoms will 1) get worse, 2) stay the same, and 3) get better.*

You can find these probabilities by adding up the probabilities at the ends of the branches of the decision tree. P(Symptoms worse) = 0.0252 + 0.0071 + 0.0973 = 0.1296 P(Symptoms the same) = 0.0048 + 0.0013 + 0.0243 + 0.3276 = 0.3580 P(Symptoms improve) = 0.5124

- *Calculate the conditional probability that the patient has a perforated appendix if the symptoms 1) get worse; 2) stay the same or 3) get* better.

By using probability definitions, we can calculate the conditional probabilities. P(E,F) = P(E | F) * P(F) P(E,F): Joint probability of E and F together P(E | F): Conditional probability of E, given F P(F): Probability of F P(E | F) = (E,F) / P(F) P(Perforation | Symptoms worse) = P(Perforation and Symptoms worse) / P(Symptoms worse) = (0.0252 + 0.0071) / 0.1296 = 0.2492 P(Perforation | Symptoms same) = P(Perforation and Symptoms same) / P(Symptoms same) = (0.0048+ 0.0013) / 0.3593 = 0.0170 P(Perforation | Symptoms improve) = P(Perforation and Symptoms improve) / P(Symptoms improve) = 0 / 0.5124 = 0

- *Calculate the conditional probability that the patient has appendicitis if 1) the symptoms get worse, 2) stay the same, or 3) get better*

Using the same probability notations as in question f, we can calculate the following probabilities: P(Appendicitis | Symptoms worse) = (0.0252 + 0.0071 + 0.0973) / 0.1296 = 1 P(Appendicitis | Symptoms same) = (0.0048 + 0.0013 + 0.0243) / 0.3580 = 0.0849 P(Appendicitis | Symptoms improve) = 0 / 0.5124 = 0