Case Study: Bayes' Theorem

> **!** Important
>
> Please note that you can download PDF and Microsoft Word versions of this case study using the links on the right.

Case 1 Description

You have a new test for detecting non-small cell lung cancer. When compared to the "Gold Standard", your new test has a sensitivity of 93% and a specificity of 98%. A patient presents with persistent cough and mediastinal lymphadenopathy. Your pre-test probability that they have non-small cell lung cancer is 0.1%.

Instructions

New non-small cell lung test
Sensitivity (true positive) = 93% (.93)

- False negative: (1-.93) = .07

Specificity (true negative) = 98% (.98)

- False positive: (1-.98) = .02

Patient presents with sore throat + lymphadenopathy:

- Pre-test probability that they have strep throat =0.001

	D+	D-	**TOTAL**
T+	93 (0.93 * 100) a [Sensitivity, true-positive]	**1,998** (0.02*99,900) b [False positive, because 1-specificity=.02]	2,091
T-	7 (0.07 * 50) c [False negative, 1-sensitivity=.07]	**97,902** (0.98 * 99,900) d [Specificity, true negative]	97,909
TOTAL	100	99,900	**100,000**

- *Using Bayes Formula, calculate the post-test probability of disease. In other words, suppose your patient tests positive on the screening test. What is the probability that they actually have small cell lung cancer? Interpret your result (You can check your answer by calculating PPV in a 2X2 table)*

PV(+) = .93*.001 / (.93*.001 + .02*0.999)
= .0445

PPV (to check) = pr(D+|T+) = the proportion of people with a positive test who have the disease = 93/2,091 = 0.0445 = 4.45%

[[Positive predictive value (PPV, proportion of people with a positive test who have the disease: a/(a+b)]]

- *Using Bayes Formula, calculate the post-test probability of non-disease. In other words,* *suppose your patient tests negative on the screening test. What is the probability that they are actually disease-free? Interpret your result (You can check your answer by calculating NPV)*

PV(-) = .98*0.999/ (.98*0.999+ .07*0.001)
= 0.9999285

NPV = pr(D-|T-) = 97,902/97,909 = 0.9999 = 99.99% = the proportion of people with a negative test who don't have the disease

[[Negative predictive value (NPV, proportion of people with a negative test who don't have the disease: d/(c+d)]]

- *Post-test probability of lung cancer if you have a negative test*

1-NPV = 1- [97,902/97,907] = **0.0000715** or **0.0001** OR if students took 1-.999 instead of .9999 from the above, then they would get **0.001** or **.01** if they took 1-0.99

OR (D+|T-) = 7/97,909

- *Post-test probability of having a noncancerous infection if you have a positive test.*

Table 1: 2x2 Table for Case 1

	D+	D-	Total
T+			
T-			
Total			100,000

1-PPV = 1- 0.0445 = 95.55%
OR (D- | T+) = 1,998 / 2,091 = 0.9555 = 95.55%

Case 2 Description

Suppose you add a confirmatory test. This test is only performed on patients with a positive result on the initial screening test (2,091 individuals from case above above) and works by a different mechanism. This test has 88% sensitivity and 99.9% specificity.

Instructions

Presuming the test was just given to patients with a positive initial screening test, please calculate:

Confirmatory test, only performed on patients with (+) test on the initial screening test

- Sensitivity (true positive) = 0.88
- Specificity (true negative) = 0.999
- Patients with a (+) rapid screening test: 2,091

	D+	D-	**TOTAL**
T+	81.84 (93 * 0.88) a	1.998 (1,998*0.001) b [0.005 is the false positive rate, 1-0.995]	83.838
T-	11.16 (93 * 0.12) c [0.11 is the false negative rate, 1-0.89]	1996.002 (1,998*0.999) d	2007.162
TOTAL	93	1,998	2,091

- *Post-test probability of noncancerous infection if you have a positive test*

P(D-|T+) = 1.998/83.838 = 0.0238 = 2.38%
Or depending on rounding and the 1-PPV, could be around .03

- *Post-test probability of lung cancer if you have a negative test*

P(D+|T-) = 11.16 / 2007.162 = 0.00556 = 0.556%
Or could be .01 if rounding

- *Are using the 2 sequential tests a good strategy for detecting non-small cell lung cancer? Why or why not?*

P(D+|T+) = 81.84/83.838 = 98% (before it was 4.45%) P(D-|T-) = 1996.002/2007.162 = 99.4% (before it was 99%)

Now you have a much better chance of correctly identifying patients with non-small cell lung cancer. Applying both tests has made you have fewer false positives – although it is not perfect.

Table 2: 2x2 Table for Case 1

	D+	D-	Total
T+			
T-			
Total			2,091